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Disturbance Observer-Based Sliding Mode Control of TORA System for Floating Wind Turbines

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***Abstract*— To anti-vibration control of floating wind turbines, the underactuated translational Oscillator with Rotating Actuator (TORA) consisting of one unactuated translational cart and one actuated rotor is adapted in the nacelle of floating wind turbine acting as a damper. We analyze the effect of new proposed disturbance observer-based sliding mode control (SMC) law on nonlinear TORA system suffering vibration of the floating wind turbine as unknown exogenous disturbance. For this we design a new sliding mode surface, which is a combination of traditional, integral and terminal sliding mode surfaces along with the output of disturbance observer for canceling the effect of above-mentioned disturbance. A complete theoretical proof with simulation result is described here, for the feasibility of this new disturbance observer-based SMC.**

I. INTRODUCTION

Underactuated system is one of the category of mechanical control systems, whose number of configuration variables are greater than its control inputs. As compare to fully actuated systems, underactuated system have less number of control inputs. Due to this property, they become more attractive for energy saving, reducing the cost factor and increasing the system flexibility [1]. Underactuated system become an interested area among the researcher since last three decades, some of them are: Furuta Pendulum, inverted pendulum, Pendubot, 2D-TORA, Acrobot, Aircraft, Vertical Take-Off and Landing (VTOL), Three-Phase Voltage-Based PWM Rectifier, Underwater Robot [2]–[5].

The Translational Oscillators with Rotating Actuator (TORA) system is the combination of two components, first one is unactuated translational oscillation cart and second one is an actuated eccentric rotational proof mass. The main control target of TORA system is to control the angular position of the rotor by using control input torque, which is acting on the rotor and along with it to stabilize the translational position of the cart around its equilibrium. This underactuated (TORA) system was basically planned to simplify the dual-spin spacecraft model for capturing the resonance phenomenon and achieving the successful despin maneuver. Afterward theoretically and practically, it was treating as a testbed for nonlinear control system [6]–[8]. In [9], the authors described the structural control of offshore floating wind turbines in detail, in which they control the inner part of nacelle, which consist on tuned mass spring-damper (TMD), by using controlled force actuator and called it hybrid mass damper (HMD). The TORA system is an active mass damper (AMD) system in which role of the actuator is to provide the both restoring and damping forces on the mass, so that's why TORA system can be replace with HMD inside the nacelle of floating wind turbine as an active control system, as shown in Fig. 1.

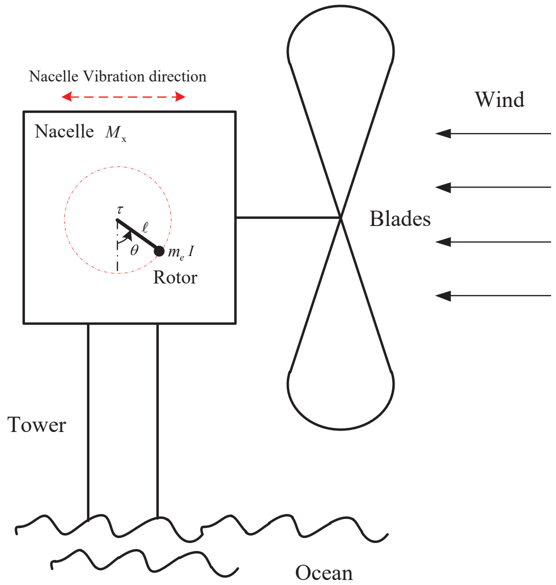

Fig. 1. Schematic of underactuated TORA as a damper in the nacelle of a floating wind turbine.

We know that disturbance generates the vibrational effect into the devices and they can be either measurable or unmeasurable. The nacelle of offshore floating wind turbines are always facing different kind of disturbances like wind disturbances, sea hurricanes or earthquakes [10], [11]. Due to the involvement of uncertainties or unmodelled dynamics into the disturbance, we can divide it into matched disturbance and second one is in mismatched disturbance. In the input states portion [12], matched disturbance will appeared while on the other portion [13], mismatched disturbance will occur. Park et al. [14], discussed the LMI based approach to control linear systems with disturbances and uncertainties. In [15], Flexible backstepping design for disturbance attenuation and tracking for nonlinear benchmark system (TORA) was presented. Tanaka et al. [16], presents decay rate and disturbance rejection from TORA system by using

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fuzzy regulator and fuzzy observer design via LMIs. Stability control of TORA affected by the environmental disturbances like an earthquake is discussed [17]. Previously different researchers works on rejection of disturbances by introducing disturbance observer (DO) and disturbance observer-based control law (DOBC) [18], [19]. In [20], [21], Choi and Chan shows that sliding mode control is not sensitive to matched disturbances while sensitive to mismatched uncertainties. In [22], Levant discuss the exact differentiation through sliding mode technique . The singularity and undesired overshoot problems were faced while developing the SMC [23] and Integral SMC [24] with respect to disturbances. To overcome the singularity issue, terminal sliding mode control and non-singular terminal sliding mode control were introduced [25], [26]. Due to finite-time stability, enhanced exogenous disturbance observer cannot be integrated with terminal or non-singular terminal sliding mode control while the latter is asymptotical stable.

In this paper, a nonlinear TORA affected system due to unknown exogenous mismatched disturbance is under observation, for this we propose a new sliding mode control scheme to minimize the oscillation of TORA system, which is used in the nacelle of the offshore floating wind turbine. The proposed sliding surface consists of traditional, integral and terminal sliding mode surfaces along with the output of disturbance observer for canceling the effect of harmonic disturbances. Firstly, in part II we presents the dynamics of TORA system and then a modified proof of disturbance observer-based SMC with stability analysis is described in part III. The exogenous disturbance observer for TORA system is presented in part IV. The last part V and VI shows the simulation results with description and conclusion respectively.

II. Dynamics of TORA System

The dynamics of underactuated translational oscillator with a rotational actuator (TORA) is shown in Fig. 2, which includes on, one cart of mass M_x with one eccentric rotational proof mass m_e.

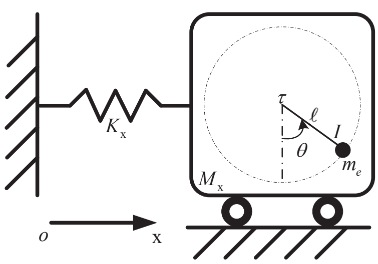

Fig. 2. TORA Model

This cart is attached inside the wall of the nacelle with a spring of stiffness k_x and has one dimensional translational motion. The eccentric rotational proof mass m_e having a moment of inertia I about its center of mass, which is attached with the cart from a point τ with a distance ℓ, where τ is the control input, applied to the proof mass. As the movement of the system is along horizontal plan, so for this we don't consider gravitational force. Let x and $\dot{\mathrm{x}}$ shows the translation position and velocity of the cart respectively and $\theta, \dot{\theta}$ shows the rotational proof mass position and its velocity. In [7], the dynamics of TORA system without disturbance force is extract by using Euler-Lagrange equations of motion, as follows

$$\begin{cases} (M_x + m_e)\ddot{\mathrm{x}} + m_e\ell\cos\theta\ddot{\theta} - m_e\ell\sin\theta\dot{\theta}^2 + k_x\mathrm{x} = 0 \\ m_e\ell\cos\theta\ddot{\mathrm{x}} + \left(m_e\ell^2 + I\right)\ddot{\theta} = \tau \end{cases} \tag{1}$$

We can rewrite TORA system (1), by selecting the system states $x = [\mathrm{x}, \dot{\mathrm{x}}, \theta, \dot{\theta}]^{\mathrm{T}} = [x_1, x_2, x_3, x_4]^{\mathrm{T}}$ and $\tau = \mathrm{u}$, as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \mathrm{f}_1(x) + \mathrm{b}_1(x)\mathrm{u} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \mathrm{f}_2(x) + \mathrm{b}_2(x)\mathrm{u} \end{cases} \tag{2}$$

where

$$\begin{aligned} \mathrm{f}_1(x) &= \frac{(m_e\ell^2+I)(m_e\ell\sin x_3 x_4^2 - k_x x_1)}{(M_x+m_e)(m_e\ell^2+I) - (m_e\ell\cos x_3)^2} \\ \mathrm{b}_1(x) &= \frac{-m_e\ell\cos x_3}{(M_x+m_e)(m_e\ell^2+I) - (m_e\ell\cos x_3)^2} \\ \mathrm{f}_2(x) &= \frac{m_e\ell\cos x_3(k_x x_1 - m_e\ell\sin x_3 x_4^2)}{(M_x+m_e)(m_e\ell^2+I) - (m_e\ell\cos x_3)^2} \\ \mathrm{b}_2(x) &= \frac{(M_x+m_e)}{(M_x+m_e)(m_e\ell^2+I) - (m_e\ell\cos x_3)^2} \end{aligned}$$

III. Observer-Based SMC Design

To fade out the effect of unknown exogenous mismatched disturbances, which interrupt our system. We follow the procedure [18], in which the controller and the disturbance observer are interconnected with each other. For this we decomposed system (2) into two second-order nonlinear subsystems with edition of mismatched disturbance and also show the desired output of the respective subsystems, as follows

$$\begin{cases} \dot{x}_1 = x_2 + w_x \\ \dot{x}_2 = \mathrm{f}_1(x) + \mathrm{b}_1(x)\mathrm{u} \\ y \ = x_1 \end{cases} \tag{3}$$

and

$$\begin{cases} \dot{x}_3 = x_4 + w_\theta \\ \dot{x}_4 = \mathrm{f}_2(x) + \mathrm{b}_2(x)\mathrm{u} \\ y \ = x_3 \end{cases} \tag{4}$$

where w_x and w_θ are the disturbances along the cart and the rotational proof mass subsystems respectively. To follow the procedure of hierarchal SMC, we assume that control input $\mathrm{u} = \mathrm{u}_x$ will control subsystem (3) and similarly $\mathrm{u} = \mathrm{u}_\theta$ will control subsystem (4). Firstly, for subsystem (3), we have chosen a new sliding mode surface σ_x, as follows

$$\sigma_x = c_1x_1 + c_2x_2 + c_2\hat{w}_x + c_3\int x_1 + \frac{1}{\gamma_x}(x_2 + \hat{w}_x)^{\frac{s_x}{t_x}} \tag{5}$$

where γ_x, c_1, c_2 and c_3 are positive constants, s_x and t_x are positive odd integers which can be designed as $1 < s_x/t_x < 2$, while $\hat{w}_x$ represents the estimated disturbance. For simplicity, in standard form, we can write equation (5), as follows

$$\sigma_x = c_1\tilde{x}_1 + c_2\tilde{x}_2 + c_3\int \tilde{x}_1 + \frac{1}{\gamma_x}\tilde{x}_2^{\frac{s_x}{t_x}} \tag{6}$$

where

$$\tilde{x}_1 = x_1$$
$$\tilde{x}_2 = x_2 + \hat{w}_{\mathrm{x}}$$

To converge sliding mode surface to zero according to sliding mode theory, we will define Lyapunov candidate, as follows

$$V_{\mathrm{x}}(t) = \sigma_{\mathrm{x}}^2/2 \tag{7}$$

By taking derivative with respect to time yields

$$\dot{V}_{\mathrm{x}}(t) = \sigma_{\mathrm{x}}\dot{\sigma}_{\mathrm{x}} \tag{8}$$

After taking derivative of equation (5) and invoking equation (3), $\tilde{x}_2$ and $\eta_{\mathrm{x}} = \frac{s_{\mathrm{x}}}{t_{\mathrm{x}}\gamma_{\mathrm{x}}}$, equation (8) can be written as

$$\dot{V}_{\mathrm{x}}(t) = \sigma_{\mathrm{x}}\{c_1(x_2 + w_{\mathrm{x}}) + c_3x_1 + (c_2 + \eta_{\mathrm{x}}\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1})(\mathrm{f}_1(x) + \mathrm{b}_1(x)\mathrm{u}_{\mathrm{x}} + \dot{\hat{w}}_{\mathrm{x}})\} \tag{9}$$

We can evaluate the derivative of estimated disturbance by using first-order sliding mode differentiator [22] such that, $\dot{\hat{w}}_{\mathrm{x}} = \xi_{\hat{w}_{\mathrm{x}}} + \varepsilon_{\hat{w}_{\mathrm{x}}}$, where $\varepsilon_{\hat{w}_{\mathrm{x}}}$ is a small constant value and $\xi_{\hat{w}_{\mathrm{x}}}$ is, as follows

$$\begin{cases} \xi_{\hat{w}_{\mathrm{x}}} = -g_0|\hat{w}_{\mathrm{x}} - w_{\mathrm{x}}|^{\frac{1}{2}}\mathrm{sign}(\hat{w}_{\mathrm{x}} - w_{\mathrm{x}}) + \Gamma_0 \\ \dot{\Gamma}_0 = -g_1\mathrm{sign}(\hat{w}_{\mathrm{x}} - w_{\mathrm{x}}) \end{cases} \tag{10}$$

where $g_0 > 0$ and $g_1 > 0$ are the constants and Γ_0 is an additional state with initial value $\Gamma_0(0) = 0$. A new disturbance observer-based sliding mode control law is designed as:

$$\mathrm{u}_{\mathrm{x}} = -\mathrm{b}_1{}^{-1}(x)\left(\mathrm{f}_1(x) + \frac{c_1\tilde{x}_2 + c_3x_1}{c_2 + \eta_{\mathrm{x}}\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1}} + \xi_{\hat{w}_{\mathrm{x}}} + G_{\mathrm{x}}\mathrm{sgn}(\sigma_{\mathrm{x}})\right) \tag{11}$$

where the gain constant is $G_{\mathrm{x}} > 0$ and by substituting u_{x} in equation (9) yields

$$\begin{aligned}\dot{V}_{\mathrm{x}}(t) &= \sigma_{\mathrm{x}}[c_1(x_2 + w_{\mathrm{x}}) + c_3x_1 + (c_2 + \eta_{\mathrm{x}}\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1})\{\mathrm{f}_1(x) \\ &\quad + \mathrm{b}_1(x)\left(-\mathrm{b}_1{}^{-1}(x)(\mathrm{f}_1(x) + (c_2 + \eta_{\mathrm{x}}\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1})^{-1}(c_1\tilde{x}_2 \right. \\ &\quad \left. + c_3x_1) + \xi_{\hat{w}_{\mathrm{x}}} + G_{\mathrm{x}}\mathrm{sgn}(\sigma_{\mathrm{x}}))\right) + \xi_{\hat{w}_{\mathrm{x}}} + \varepsilon_{\hat{w}_{\mathrm{x}}}\}] \\ &= \sigma_{\mathrm{x}}\{c_1\tilde{w}_{\mathrm{x}} - (c_2 + \eta_{\mathrm{x}}\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1})(G_{\mathrm{x}}\mathrm{sgn}(\sigma_{\mathrm{x}}) - \varepsilon_{\hat{w}_{\mathrm{x}}})\} \\ &\leq |\sigma_{\mathrm{x}}|\varepsilon_0 - G_{\mathrm{x}}(c_2 + \eta_{\mathrm{x}}\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1})|\sigma_{\mathrm{x}}| + (c_2 \\ &\quad + \eta_{\mathrm{x}}\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1})\varepsilon_1|\sigma_{\mathrm{x}}| \\ &\leq -|\sigma_{\mathrm{x}}|\{(G_{\mathrm{x}}c_2 - c_2\varepsilon_1) + \tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1}(G_{\mathrm{x}}\eta_{\mathrm{x}} - \eta_{\mathrm{x}}\varepsilon_1)\} \\ &\quad + |\sigma_{\mathrm{x}}|\varepsilon_0 \\ &\leq -(P + Q\,\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1} - \varepsilon_0)|\sigma_{\mathrm{x}}| \\ &\leq -(P + Q\,\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1} - Y)|\sigma_{\mathrm{x}}| \end{aligned} \tag{12}$$

where $\tilde{w}_{\mathrm{x}} = e_{w_{\mathrm{x}}} = w_{\mathrm{x}} - \hat{w}_{\mathrm{x}}$ is an approximated error such that $\tilde{w}_{\mathrm{x}} < \varepsilon_0$ with $\varepsilon_0 > 0$ and $|\varepsilon_{\hat{w}_{\mathrm{x}}}| < \varepsilon_1$ with $\varepsilon_1 > 0$, while $P = G_{\mathrm{x}}c_2 - c_2\varepsilon_1$, $Q = G_{\mathrm{x}}\eta_{\mathrm{x}} - \eta_{\mathrm{x}}\varepsilon_1$ and $Y > \varepsilon_0$. We can design a constant for Lyapunov stability such as $P^{\circ} > (P + Q\,\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1} - Y)$. The states will go to sliding surface efficiently by choosing sufficient high gain constants (P and Q). As we mention before that ratio of s_{x} and t_{x} will be in between 1 and 2, so we can say that $\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1} > 0$ for nonzero x_2. Lets assume that $H(x_2, \hat{w}_{\mathrm{x}}) = P + Q\,\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1} - Y$ such that $H(x_2, \hat{w}_{\mathrm{x}}) > 0$, yields

$$\dot{V}_{\mathrm{x}}(t) = \sigma_{\mathrm{x}}\dot{\sigma}_{\mathrm{x}} \leq -H(x_2, \hat{w}_{\mathrm{x}})|\sigma_{\mathrm{x}}| \tag{13}$$

Therefore, for case $x_2 \neq 0$, Lyapunov stability is ensured. The nonlinear system states will reach to sliding surface $\sigma_{\mathrm{x}} = 0$ asymptotically. It can be verify during the $x_2 = 0$ case. The equation (3), by invoking equation (11) yields

$$\dot{x}_2 = -\frac{c_1\tilde{x}_2 + c_3x_1}{c_2 + \eta_{\mathrm{x}}\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1}} - \xi_{\hat{w}_{\mathrm{x}}} - G_{\mathrm{x}}\mathrm{sgn}(\sigma_{\mathrm{x}}) \tag{14}$$

By substituting $x_2 = 0$ in $\tilde{x}_2$, it will gives $\tilde{x}_2 = \hat{w}_{\mathrm{x}}$, so invoking it with equation (14), yields

$$\dot{x}_2 = -\left(\frac{c_1\hat{w}_{\mathrm{x}} + c_3x_1}{c_2 + \eta_{\mathrm{x}}\hat{w}_{\mathrm{x}}^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1}} + \xi_{\hat{w}_{\mathrm{x}}} + G_{\mathrm{x}}\mathrm{sgn}(\sigma_{\mathrm{x}})\right) \tag{15}$$

Lets defining a bound D such that $D \geq \frac{c_1\hat{w}_{\mathrm{x}} + c_3x_1}{c_2 + \eta_{\mathrm{x}}\hat{w}_{\mathrm{x}}^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1}} + \xi_{\hat{w}_{\mathrm{x}}}$ and invoking in equation (15) yields

$$\dot{x}_2 \leq -\{D + G_{\mathrm{x}}\mathrm{sgn}(\sigma_{\mathrm{x}})\} \tag{16}$$

Therefore to follow the Feng et al. [26] procedure, by choosing large $G_{\mathrm{x}} > 0$, we can say that $\dot{x}_2 \leq -G_{\mathrm{x}}$ for surface $\sigma_{\mathrm{x}} > 0$ and $\dot{x}_2 \geq G_{\mathrm{x}}$ for surface $\sigma_{\mathrm{x}} < 0$, which means that the state x_2 is equal to zero is not an attractor so we can consider the near surrounding region of the state x_2 equal to zero, such as $\delta > |x_2|$, where $\delta > 0$ ensure the Lyapunov stability discussed in [18], such as $\dot{V}_e(t) = -\delta e_{w_{\mathrm{x}}}^{\mathrm{T}} e_{w_{\mathrm{x}}}$, hence still the above mentioned condition will exists i.e., $\sigma_{\mathrm{x}} > 0$ and $\sigma_{\mathrm{x}} < 0$ for $x_2 \leq -G_{\mathrm{x}}$ and $x_2 \geq G_{\mathrm{x}}$ respectively. Therefore, for $\sigma_{\mathrm{x}} > 0$ the trajectory will be crossing from $x_2 = \delta$ to $x_2 = -\delta$ and similarly for case $\sigma_{\mathrm{x}} < 0$. We can conclude that from equation (12) the switching surface $\sigma_{\mathrm{x}} > 0$ will reached asymptotically to outside the surrounding area of $\delta < x_2$. From this we can say that when the switching surface is achieved, states will go to their equilibrium points asymptotically.

A. Stability Analysis For Disturbance Observer-Based SMC

We present the stability analysis for disturbance observer-based sliding mode control by defining a Lyapunov function, which we can write as

$$V(t) = V_{\mathrm{x}}(t) + V_e(t) \tag{17}$$

The Lyapunov candidate function $V_{\mathrm{x}}(t)$ represents the stability of sliding mode control law, while $V_e(t)$ shows the stability of disturbance observer [18]. Take the derivative of equation (17) w.r.t time and invoking the respective values, such as

$$\dot{V}(t) \leq -\left((P + Q\,\tilde{x}_2^{\frac{s_{\mathrm{x}}}{t_{\mathrm{x}}}-1} - Y)|\sigma_{\mathrm{x}}| + \delta e_{w_{\mathrm{x}}}^{\mathrm{T}} e_{w_{\mathrm{x}}}\right) \tag{18}$$

In their respective sections, the stability of the functions involved in (18) has been proved, like as stability of new sliding mode control is derived and presented in equation (12) and (16), while in [18], the Lyapunov stability of disturbance observer was described. The surface become zero i.e., $\sigma_\mathrm{x} = 0$, when the sliding mode surface is approached, by invoking it in equation (5) yields

$$\sigma_\mathrm{x} = c_1 x_1 + c_2 x_2 + c_2 \hat{w}_\mathrm{x} + c_3 \int x_1 + \frac{1}{\gamma_\mathrm{x}} (x_2 + \hat{w}_\mathrm{x})^{\frac{s_\mathrm{x}}{t_\mathrm{x}}} = 0 \tag{19}$$

From equation (3) and e_{w_x}, we can derived that

$$x_2 = \dot{x}_1 - w_\mathrm{x}$$

$$\ddot{x}_1 + \frac{c_1 \dot{x}_1 + c_3 x_1}{c_2 + \frac{s_\mathrm{x}}{t_\mathrm{x}\gamma_\mathrm{x}} (\dot{x}_1 - e_{w_\mathrm{x}})^{\frac{s_\mathrm{x}}{t_\mathrm{x}} - 1}} = \dot{e}_{w_\mathrm{x}} \tag{20}$$

Its show that in finite time if the system reached to the sliding surface $\sigma_\mathrm{x} = 0$ then the state can be reached to the desired equilibrium point asymptotically, where disturbance error has a constant steady state value, i.e., $\lim_{t\to\infty} \dot{e}_{w_\mathrm{x}} = 0$. Furthermore, by using the property $\sigma_\mathrm{x}\dot{\sigma}_\mathrm{x} < 0$ of Lyapunov candidate $V(t)$ along with condition $|\sigma_\mathrm{x}|^\mathrm{v} \leq 1 + |\sigma_\mathrm{x}|$, where $0 < \mathrm{v} < 1$, we can conclude that states x_1 and $\tilde{x}_2 = x_2 + \hat{w}$ will remain on a manifold and not escape from it. So, its proved that through choosing suitable gain values, we can make disturbance observer based sliding mode control law, asymptotically stable. Similarly, same procedure will apply for second subsystem (4), to obtain sliding mode control law u_θ by using new sliding mode surface σ_θ, as follows

$$\sigma_\theta = c_4 x_3 + c_5 x_4 + c_5 \hat{w}_\theta + c_6 \int x_3 + \frac{1}{\gamma_\theta} (x_4 + \hat{w}_\theta)^{\frac{s_\theta}{t_\theta}} \tag{21}$$

$$\mathrm{u}_\theta = -\mathrm{b}_2^{-1}(x) \left(\mathrm{f}_2(x) + \frac{c_4 \tilde{x}_4 + c_6 x_3}{c_5 + \eta_\theta \tilde{x}_4^{\frac{s_\theta}{t_\theta} - 1}} + \xi_{\hat{w}_\theta} + G_\theta \mathrm{sgn}(\sigma_\theta) \right) \tag{22}$$

where γ_θ, c_4, c_5 and c_6 are positive constants, s_θ and t_θ are positive odd integers which can be designed as $1 < s_\theta / t_\theta < 2$, while $\hat{w}_\theta$ represents the estimated disturbance, $G_\theta > 0$ represents gain constant, $\eta_\theta = \frac{s_\theta}{t_\theta \gamma_\theta}$ and $\xi_{\hat{w}_\theta}$ is, as follows

$$\begin{cases} \xi_{\hat{w}_\theta} = -g_2 |\hat{w}_\theta - w_\theta|^{\frac{1}{2}} \mathrm{sign}(\hat{w}_\theta - w_\theta) + \Gamma_1 \\ \dot{\Gamma}_1 \ = -g_3 \mathrm{sign}(\hat{w}_\theta - w_\theta) \end{cases} \tag{23}$$

where $g_2 > 0$ and $g_3 > 0$ are the constants and Γ_1 is an additional state with initial value $\Gamma_1(0) = 0$. Now we will follow the hierarchical SMC method [7], to control the whole TORA system with one control input. For this we can define total sliding mode surface, as follows

$$\sigma_T = \sigma_x + \sigma_\theta \tag{24}$$

Let us define a Lyapunov function for stability analysis of combined subsystem surfaces of TORA system, such as

$$V_{Total}(t) = \frac{{\sigma_T}^2}{2} \tag{25}$$

By taking the derivative of equation (25), as follows

$$\begin{cases} \dot{V}_{Total}(t) = \sigma_T \dot{\sigma}_T \\ \qquad = \sigma_T \left[I_x + I_\theta + J_x \mathrm{f}_1(x) + J_\theta \mathrm{f}_2(x) \right. \\ \qquad + (J_x \mathrm{b}_1(x) + J_\theta \mathrm{b}_2(x)) \mathrm{u} + J_x \xi_{\hat{w}_x} \\ \qquad \left. + J_\theta \xi_{\hat{w}_\theta} + J_x \varepsilon_{\hat{w}_x} + J_\theta \varepsilon_{\hat{w}_\theta} \right] \end{cases} \tag{26}$$

where $I_x = c_1 \tilde{x}_2 + c_3 x_1$, $I_\theta = c_4 \tilde{x}_4 + c_6 x_3$, $J_x = c_2 + \eta_x \tilde{x}_2^{\frac{s_x}{t_x} - 1}$, $J_\theta = c_5 + \eta_\theta \tilde{x}_4^{\frac{s_\theta}{t_\theta} - 1}$. Now we will design disturbance observer-based sliding mode control law u, which has a combine effect of $\mathrm{u_x}$ and u_θ, in such a way that our total surface goes to zero asymptotically, as follows

$$\begin{aligned} \mathrm{u} = -(J_x \mathrm{b}_1(x) + J_\theta \mathrm{b}_2(x))^{-1} \left[I_x + I_\theta + J_x \mathrm{f}_1(x) \right. \\ \left. + J_\theta \mathrm{f}_2(x) + J_x \xi_{\hat{w}_x} + J_\theta \xi_{\hat{w}_\theta} + G_T \mathrm{sgn}(\sigma_T) \right] \end{aligned} \tag{27}$$

where gain constant $G_T > 0$. The remaining procedure and description will be the same, like Lyapunov stability for individual sliding mode surface σ_x. So after substituting and calculating, we will get, such as

$$\dot{V}_{Total}(t) = -H(x, \hat{w}) |\sigma_T| \leq 0 \tag{28}$$

where $H(x, \hat{w}) > 0$ hence, the equation (28) shows the proof of Lyapunov stability for combined subsystem surfaces of TORA system.

IV. Mismatched Exogenous Disturbance Observer For TORA System

The general form of exogenous disturbance observer for nonlinear system is described in [18]. For subsystem (3), we take $p_\mathrm{x}(x) = K_\mathrm{x} x_1$, where $K_\mathrm{x} = \begin{bmatrix} k_1 & k_2 \end{bmatrix}^\mathrm{T}$ in such a way that k_1, k_2 are the design constants and x_1 is the desired output state of subsystem (3). We need to choose such a value of design constants that the error dynamics of exogenous disturbance observer become asymptotically stable. We used the same exogenous disturbance but deal here as a mismatched harmonic disturbance i.e., $w(t) = \frac{1}{2}\sin(2t + 1)$ with $A = \begin{bmatrix} 0 & 2; & -2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, as explained in example study [18]. So, by using these information we can find other parameter and through invoking all the parameter values into the general exogenous disturbance observer [18], we can get following exogenous disturbance observer for subsystem (3), such as

$$\begin{cases} \dot{z}_\mathrm{x} = \begin{bmatrix} 2z_{\mathrm{x}_2} - k_1 z_{\mathrm{x}_1} + 2k_2 x_1 - k_1 k_1 x_1 - k_1 x_2 \\ -2z_{\mathrm{x}_1} - k_2 z_{\mathrm{x}_1} - 2k_1 x_1 - k_1 k_2 x_1 - k_2 x_2 \end{bmatrix} \\ \hat{\zeta}_\mathrm{x} = z_\mathrm{x} + p_\mathrm{x}(x) \\ \hat{w}_\mathrm{x} = C \hat{\zeta}_\mathrm{x} \end{cases} \tag{29}$$

where $\hat{w}_\mathrm{x} = \hat{\zeta}_{\mathrm{x}_1}$ is the estimated disturbance, which we will used in respective proposed sliding mode surface and proposed SMC. The observer estimation error can be find as $e_{w_\mathrm{x}} = w_\mathrm{x} - \hat{w}_\mathrm{x}$. Furthermore, the same procedure will be repeated by using $p_\theta(x) = K_\theta x_3$, where $K_\theta = \begin{bmatrix} k_3 & k_4 \end{bmatrix}^\mathrm{T}$ in such a way that k_3 and k_4 are the design constants and x_3 is the desired output state of subsystem (4). Then by calculating and substituting the values for remaining parameters, we can find the exogenous disturbance observer for TORA subsystem (4), as follows

$$\begin{cases} \dot{z}_\theta = \begin{bmatrix} 2z_{\theta_4} - k_3 z_{\theta_3} + 2k_4 x_3 - k_3 k_3 x_3 - k_3 x_4 \\ -2z_{\theta_3} - k_4 z_{\theta_3} - 2k_3 x_3 - k_3 k_4 x_3 - k_4 x_4 \end{bmatrix} \\ \hat{\zeta}_\theta = z_\theta + p_\theta(x) \\ \hat{w}_\theta = C\hat{\zeta}_\theta \end{cases} \tag{30}$$

where $\hat{w}_\theta = \hat{\zeta}_{\theta_3}$ is the estimated disturbance and observer estimation error will be $e_{w_\theta} = w_\theta - \hat{w}_\theta$. The simulation results for effective mismatched exogenous disturbance observer-based SMC law for TORA system with selection of appropriate values of constants and gains are shown in next section.

V. Simulation Results

To analyze the performance of presented disturbance observer-based sliding mode control law for the stabilization of nonlinear TORA system, the simulations of close-loop controlled system are performed on Matlab/Simulink.

Based on [6], the parameters used for TORA system are chosen as: $m_e = 0.096$kg, $M_x = 1.3608$kg, $k_x = 186.3$N/m, $I = 0.0002175\text{kg}\cdot\text{m}^2$, $\ell = 0.0592$m. The parameters of disturbance observer-based sliding mode control law are designed as

$$\begin{aligned} c_1 &= c_2 = c_3 = 40 \\ c_4 &= c_6 = 35 \\ c_5 &= 0.7 \\ s_x &= s_\theta = 111 \\ t_x &= t_\theta = 109 \\ \gamma_x &= 1 \\ \gamma_\theta &= 2 \\ k_1 &= k_2 = 5 \\ k_3 &= k_4 = 8 \\ g_0 &= g_1 = g_2 = g_3 = 0.001 \\ G_x &= G_\theta = 50 \\ G_T &= 100 \end{aligned}$$

The initial conditions used for controlled TORA system are $x_1(0) = 0.1$, $x_2(0) = 0$, $x_3(0) = \pi/12$, $x_4(0) = 0$ and additional variable $z(0)$=0. The simulation results of stabilized TORA system shown in Fig. 3 are examined to be the same under different initial conditions.

The efficiency of our proposed separate SMC (u_x, u_θ) can be seen from Fig. 3(a) that both the desired states of TORA system goes to equilibrium position within 9 seconds asymptotically with respect to their respective sliding mode surfaces as shown in Fig. 3(b). Similarly, from Fig. 3(c), it can be seen that the total sliding mode surface (σ_T) asymptotically goes to zero and both the desired states become stable with a very small steady state error, by using single SMC law (u), as shown in Fig. 3(d). The Fig. 3(e) shows that exogenous disturbance observer asymptotically estimate the general harmonic disturbance in same time for both cases, that is for separate SMC and for single SMC

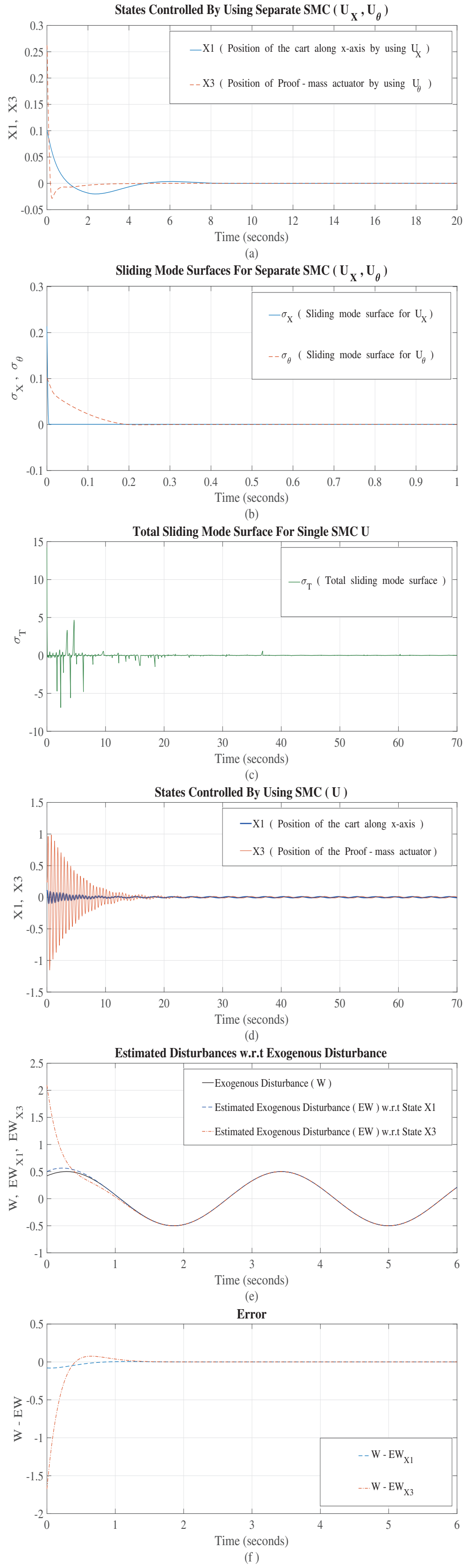

Fig. 3. A stable TORA system (a) Controlled states by using separate SMC (u_x,u_θ),(b) Sliding mode surfaces for separate SMC (u_x,u_θ), (c) Total sliding mode surface for single SMC (u), (d) States controlled by single SMC (u), (e)Estimated disturbances with respect to exogenous disturbance, (f) Error of estimated disturbances with respect to exogenous disturbance.

law. The error of estimated disturbances with respect to exogenous disturbance is shown in Fig. 3(f). In general, mostly in such cases, sliding mode control law is facing a small chattering issue, which can be reduce by using super twisting algorithm. The proposed SMC in equation (27) can be rewrite as

$$\begin{cases} \mathrm{u} = -(J_x \mathrm{b}_1(x) + J_\theta \mathrm{b}_2(x))^{-1} [I_x + I_\theta + J_x \mathrm{f}_1(x) + J_\theta \mathrm{f}_2(x) \\ \quad + J_x \xi_{\hat{w}_x} + J_\theta \xi_{\hat{w}_\theta} + G_{T_1} \sqrt{|\sigma_T|} \mathrm{sgn}(\sigma_T) - E] \\ \dot{E} \; = -G_{T_2} \mathrm{sgn}(\sigma_T) \end{cases} \tag{31}$$

where the switching gains are $G_{T_1} > 0$ and $G_{T_2} > 0$. Due to the robustness and chattering free property of the super twisting algorithm, we can improve the performance of our proposed SMC.

VI. Conclusion

The purpose of this research work is to attenuate the effect of exogenous mismatched disturbances from the nacelle of floating wind turbine by stabilizing it's inside infrastructure (TORA system). For this, we presented a modified sliding mode control scheme such that, a complete theoretically observer-based SMC design proof along with stability analysis with respect to Lyapunov function for both subsystems of TORA system is explained. Then using hierarchical SMC technique, we design a single control law to control both subsystems of TORA system. After that, we estimate the exogenous disturbance by using disturbance observer based control method. Through simulation on Matlab/Simulink, we show that by choosing appropriate constants and gains, we can make the sliding surface more attractive and effective with respect to the proposed controller design.

As traditional SMC was unable to handle mismatched disturbances and integral SMC was also facing unsatisfactory settling time and undesired overshoot in such cases. Similarly, terminal SMC due to finite-time stability cannot be integrated with exogenous disturbance observer, while the proposed SMC resolves all these issues very effectively as shown in simulation results. However, we face a small oscillation error for controlling whole TORA system by using single SMC law. We need some optimization approach to eliminate this error, which is the future work along with hardware implementation.

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